Technical Comments

Comment on "Rocket Motor with Electric Acceleration in the Throat"

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In a recent note, Rosciszewski[1] has stated: "At the critical section $M_1 = 1$, $p = 1$, $u = 0$, $du/d\xi = \infty$, and $dp/d\xi = \infty$; however, dp/du and $du^2/d\xi$ are finite." (Bars have been dropped for the sake of convenience.) The statement regarding $du/d\xi$ and $dp/d\xi$ is incorrect since such a statement may result in an infinite rise in velocity and pressure immediately downstream of the channel entrance. It will also be shown that $du^2/d\xi = 0$ at $\xi = 0$.

Since the pressure and velocity are finite throughout, the vanishing of the denominator must be accompanied by the vanishing of the numerator,[2] i.e., at the critical point,

$$\Lambda = 1 \qquad \text{or} \qquad \Lambda = \gamma/(\gamma - 1) \tag{1}$$

with the second condition holding for continuous acceleration from subsonic to supersonic speeds.

With Λ known as $\gamma/(\gamma - 1)$, $du/d\xi$ and $dp/d\xi$ are both of the form 0/0. L'Hospital's rule, applied to Eqs. (10) and (11), yields, at $\xi = 0$,

$$\frac{du}{d\xi} = \frac{1}{\gamma(\gamma - 1)} \frac{(\gamma - 1)(d\Lambda/dp) - \gamma(du/dp)}{1 - (du/dp)} \tag{2}$$

and

$$\frac{dp}{d\xi} = \frac{1}{(\gamma - 1)} \frac{1 - (\gamma - 1)[(d\Lambda/dp) - (du/dp)]}{1 - (du/dp)} \tag{3}$$

where

$$\frac{du}{dp} = -\frac{1}{2}\left\{\frac{2}{\gamma - 1} - \frac{d\Lambda}{dp} \pm \left[\left(\frac{2}{\gamma - 1} - \frac{d\Lambda}{dp}\right)^2 + \frac{4}{\gamma}\frac{d\Lambda}{dp}\right]^{1/2}\right\} \tag{4}$$

For accelerating flows $du/dp < 0$, hence if $d\Lambda/dp \geq 0$, the + sign is appropriate. On the other hand, if $d\Lambda/dp < 0$, both signs yield $du/dp < 0$. Because $du/d\xi$ is finite at $\xi = 0$, $du^2/d\xi = 0$ at $\xi = 0$.

Since $d\Lambda/d\xi$ was specified in Ref. 1, $d\Lambda/dp$ can be calculated from $d\Lambda/d\xi$ by employing Eqs. (3), (4), and the relation

$$d\Lambda/d\xi = (d\Lambda/dp)dp/d\xi \tag{5}$$

The starting values and the starting derivatives give the information needed to start the numerical integration. The integration when Λ is constant can, however, be carried out analytically.[3]

References

[1] Rosciszewski, J., "Rocket motor with electric acceleration in the throat," J. Spacecraft Rockets **2**, 278–280 (1965).

[2] Resler, E. L., Jr. and Sears, W. R., "The prospects for magneto-aerodynamics," J. Aeronaut. Sci. **25**, 235–245, 258 (1958).

[3] Oates, G. C., "Constant-electric-field and constant-magnetic-field magnetogasdynamic channel flow," J. Aerospace Sci. **29**, 231–232 (1962).

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Reply by Author to H. A. Hassan

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The author does not agree with Hassan's statement that infinite values of $du/d\xi_{\xi=0}$ and $d_P/d\xi_{\xi=0}$ must necessarily lead to $u = \infty$, $p = \infty$ downstream.

As an example, take $du/d\xi \sim \xi^{-1/2}$. Therefore, $du/d\xi_{\xi=0} = \infty$, but integrating ($u = 0$ at $\xi = 0$) one gets $u \sim \xi^{1/2}$, and u is finite for $\xi \geqslant 0$ (similar considerations are valid for p). In fact this is the type of singularity in my problem.

In my paper, gas is accelerated electrically starting from the sonic conditions and there is no need for having $\Lambda = 1$ or $\Lambda = \gamma/(\gamma - 1)$, which would be required for pure electrical acceleration from subsonic to supersonic speeds in a constant cross-sectional channel.

For $\Lambda \neq 1$ and $\Lambda \neq \gamma/(\gamma - 1)$, the quantity $du^2/d\xi_{\xi=0}$ is finite. Therefore, the starting procedure of my paper is correct, and, in addition, smooth integral curves were obtained near the singular point.

The difference of opinion comes from the fact that Hassan based his remarks on the references where the pure electric acceleration from subsonic to supersonic flow was considered. This is not the case in my paper.

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Comment on "Spinning Vehicle Nutation Damper"

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Recently, Wadleigh, Galloway, and Mathur[1] have described a nutation damper consisting of a mass sliding along a rectilinear guide. Starting from the dynamic equations of the system, curves are obtained showing optimum design parameters for this particular nutation damper.

It should be noted, however, that the equations used as a starting point are not complete. Indeed, when evaluating the torque exerted on the satellite by the sliding mass, it is necessary to consider the total acceleration of this mass instead of its relative acceleration along the guide. This yields the following equations:

$$\dot{p}(I_x + my^2) - may\dot{q} = -m(2y\dot{y}p + aypr + qry^2) \tag{1}$$

$$\dot{q}(I_y + ma^2) - may\dot{p} = pr(I_z - I_x) + m(2ap\dot{y} + pra^2 + ayqr) \tag{2}$$

$$\dot{r}(I_z + my^2) = pq(I_x - I_y) + m\{pqy^2 - 2y\dot{y}r + ay(\omega_n^2 - q^2 - r^2) + 2\lambda\omega_n a\dot{y}\} \tag{3}$$

$$\ddot{y} + 2\lambda\omega_n\dot{y} - (r^2 + p^2 - \omega_n^2)y + apq + a\dot{r} = 0 \tag{4}$$

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